

That u is a viscosity solution

We can extend the work for the heat equation to generic parabolic partial differential equations. We do this by first introducing viscosity solutions to Kolmogorov PDEs as given in Crandall & Lions *crandallionsandfurtherextended, esp.inBeck2021.SomePreliminariesWetakeworkpreviouslypioneeredbyIto1942aand applyconceptsfirstappliedinBeck2021andBHJ21.lemma* : 2.7 Let $d, m \in \mathbb{N}$, $T \in (0, \infty)$. Let $\mu \in C^{1,2}([0, T] \times \mathbb{R}^d, \mathbb{R}^d)$ and $\sigma \in C^{1,2}([0, T] \times \mathbb{R}^d, \mathbb{R}^{d \times m})$ satisfying that they have non-empty compact supports and let $S = (\mu) \cup (\sigma) \subseteq [0, T] \times \mathbb{R}^d$. Let $(\Omega, \mathcal{F}, P, (F_t)_{t \in [0, T]})$ be a filtered probability space satisfying usual conditions. Let $W : [0, T] \times \Omega \rightarrow \mathbb{R}^m$ be a standard $(F_t)_{t \in [0, T]}$ -Brownian motion, and let $X : [0, T] \times \Omega \rightarrow \mathbb{R}^d$ be an $(F_t)_{t \in [0, T]}$ -adapted stochastic process with continuous sample paths satisfying for all $t \in [0, T]$ with P -a.s. that: align $X_t = X_0 + \int_0^t \mu(s, X_s) ds + \int_0^t \sigma(s, X_s) dW_s$